Block Ciphers and CPA Security

CS/ECE 407

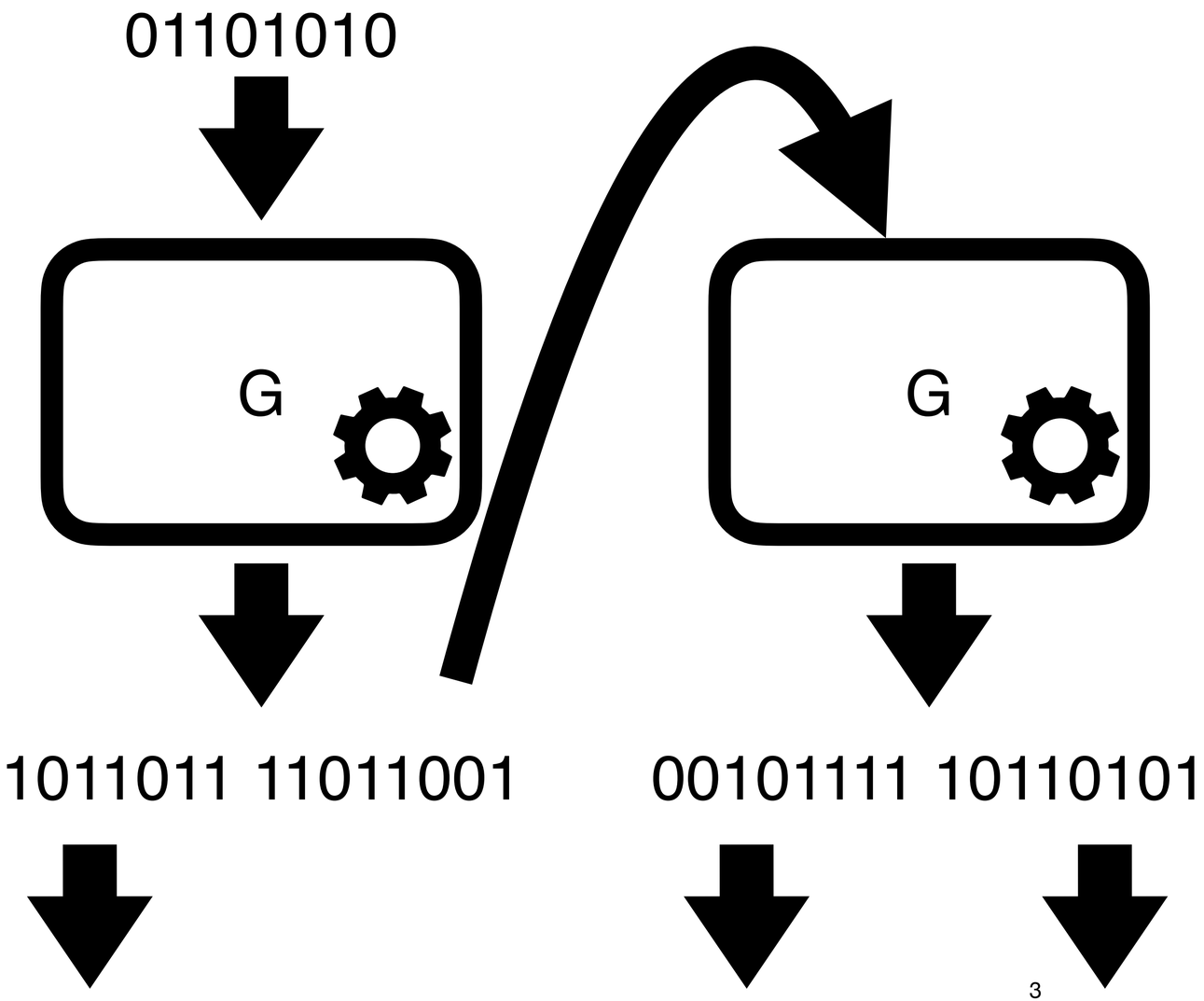
Today’s objectives

Define block ciphers

Introduce CPA Security

Understand the limitations of deterministic encryption

Stretching the output of a PRG



Call multiple times to get more randomness

What about a cryptographic primitive that generates a lot of randomness “all at once”

Alice

Bob

2^n **rows**

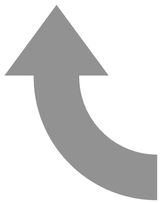
0	01101000
1	11110000
2	10001110
3	01010100
4	11011010
…	…

A pseudorandom function (PRF) allows Alice and Bob to share a huge pseudorandom table via a short key

$$F : \{0,1\}^{\lambda} \times \{0,1\}^{n} \rightarrow \{0,1\}^{m}$$

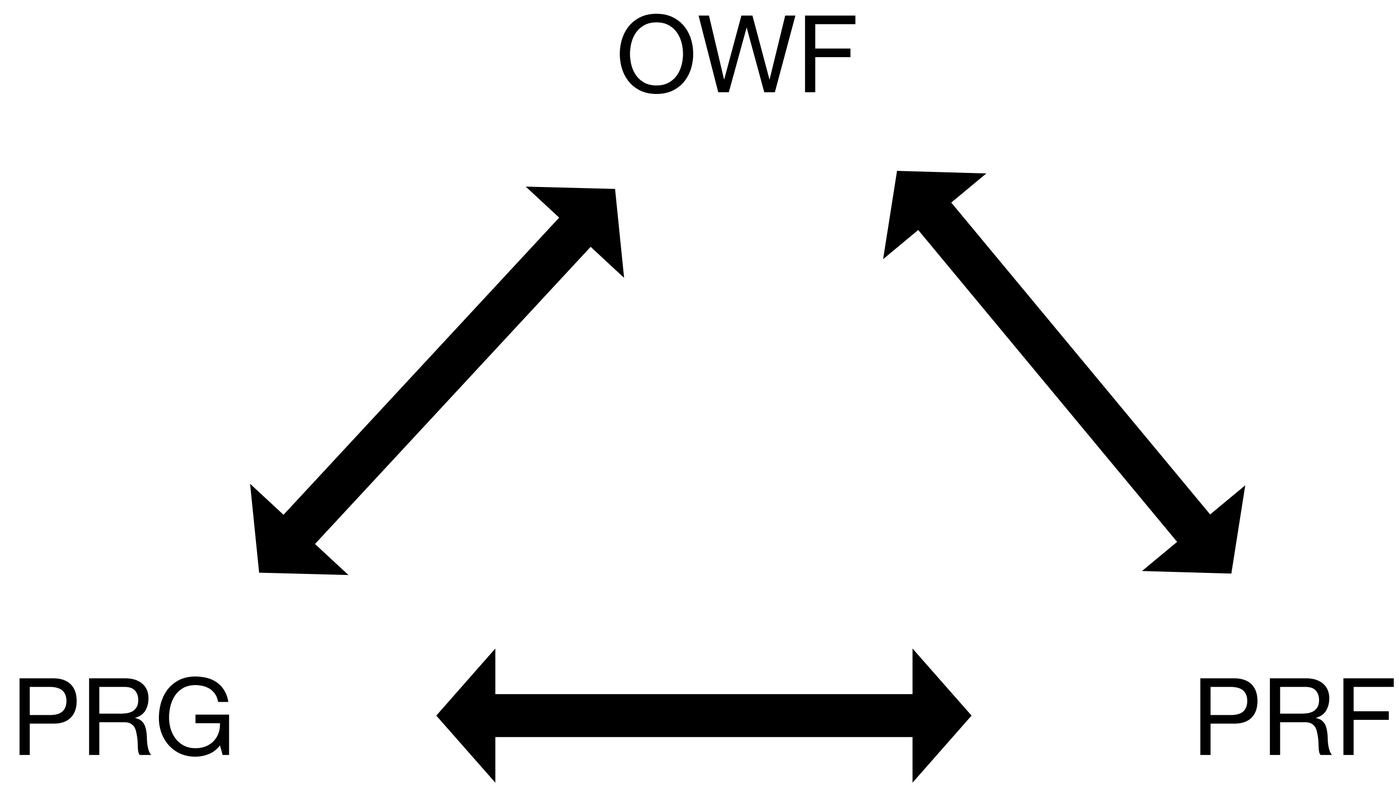
F is called a **pseudorandom function family** if the following indistinguishability holds:

$$\left\{ F(k, \cdot) \;\middle|\; k \leftarrow \{0,1\}^{\lambda} \right\} \approx \left\{ f \;\middle|\; f \leftarrow \text{uniform function from } \{0,1\}^{n} \rightarrow \{0,1\}^{m} \right\}$$



Uniformly sampling k “emulates” a huge random table

Closer to how many real-world primitives are defined



OWFs exist $\Longrightarrow$ $P \neq NP$

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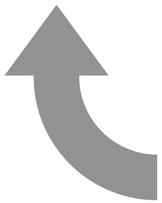

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$$\approx$$

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And there exists efficient F^{-1} s.t. $F^{-1}(k, F(k, x)) = x$

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AES is a block cipher

Every PRP with large block length is a PRF

“Switching Lemma”

$$\left\{ f \;\middle|\; f \leftarrow \text{uniform permutation from } \{0,1\}^\lambda \to \{0,1\}^\lambda \right\}$$

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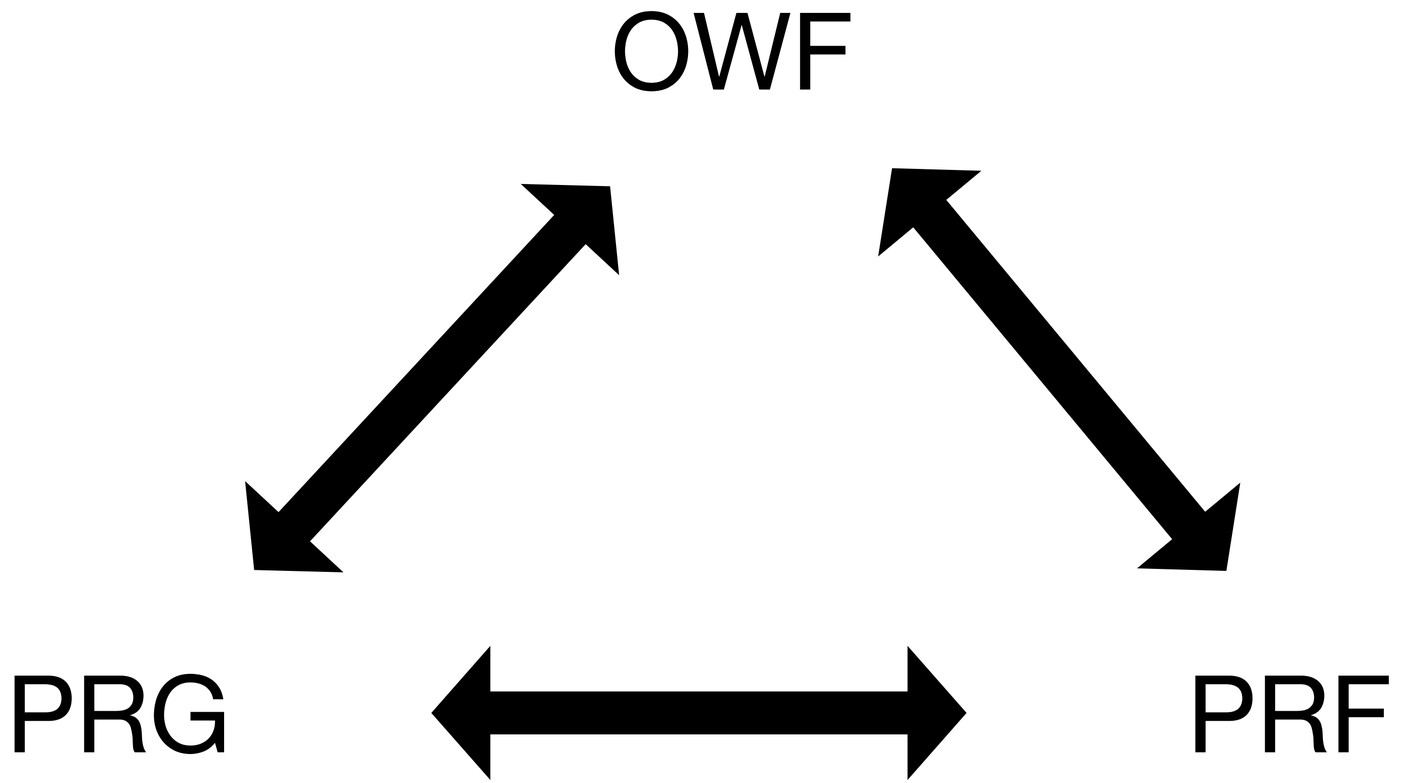
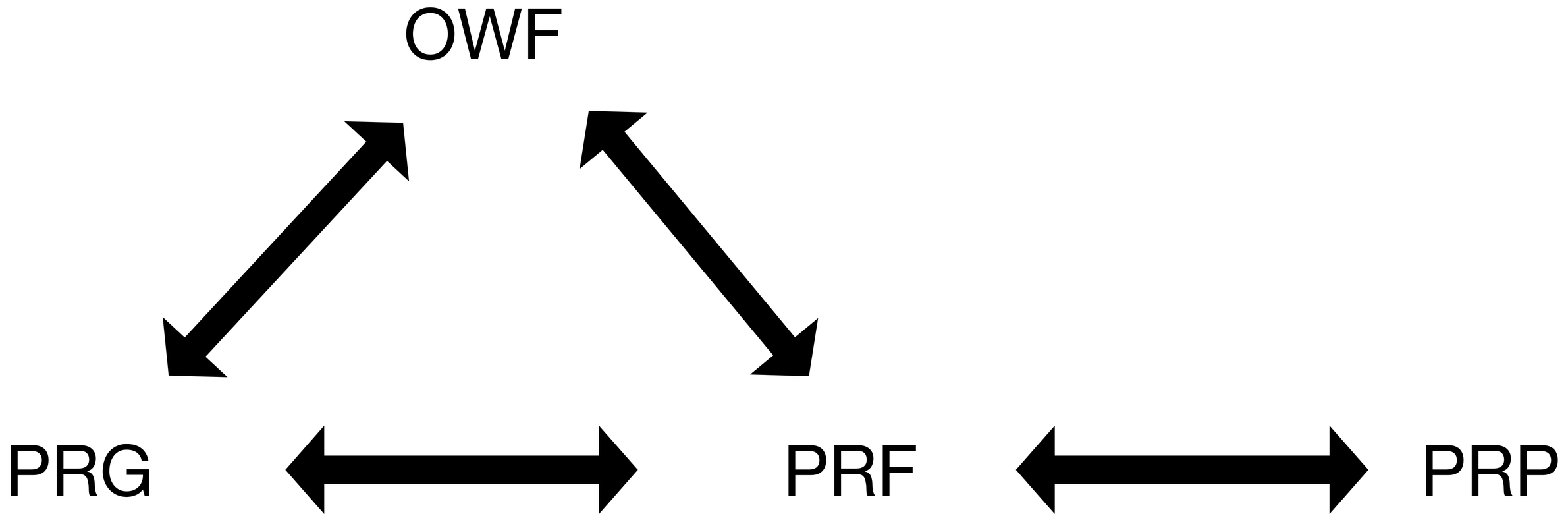
Intuition: only way to distinguish is to find a collision in function f, but collisions are rare, so polytime adversary does not have time to find one

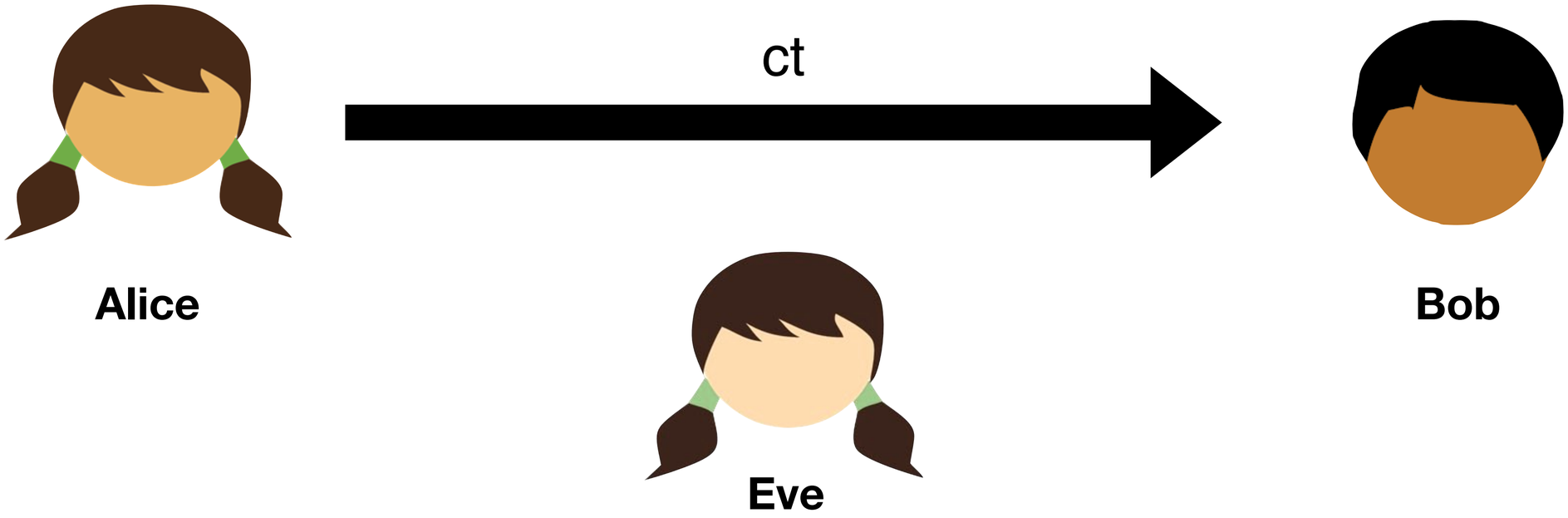

OWF
PRG
PRF


OWF
PRG
PRF
PRP

A cipher (Enc, Dec) has **one-time security** if:

For every message $m \in \mathrm{M}$:

$$\left\{ c \;\middle|\; \begin{array}{l} k \leftarrow K \\ c = Enc(k, m) \end{array} \right\} \approx \left\{ c \;\middle|\; c \leftarrow C \right\}$$



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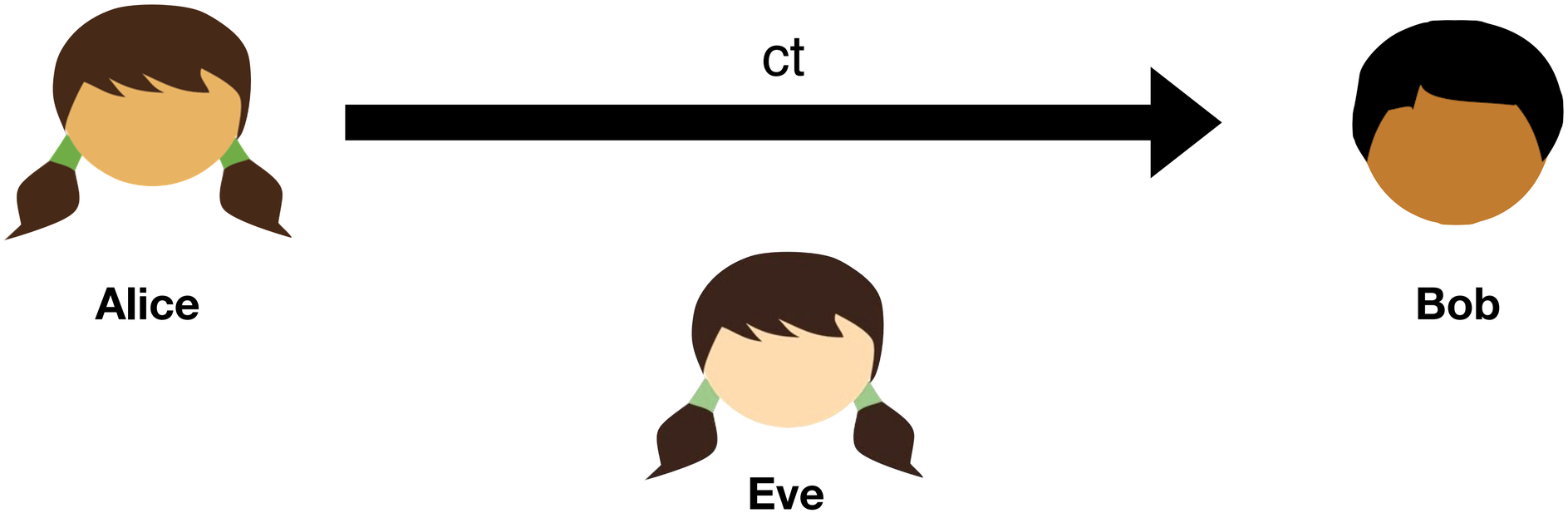
How can we implement (Enc, Dec) with a block cipher?



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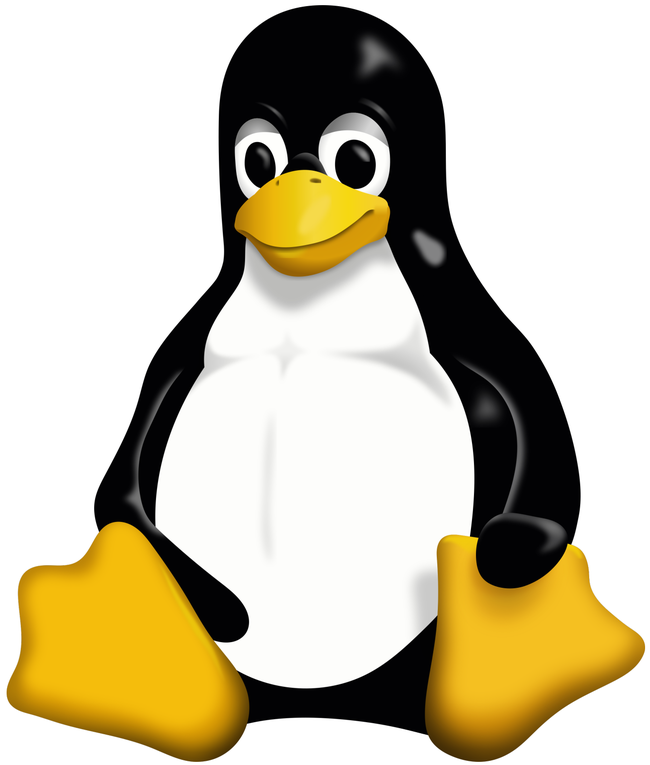
What if Alice/Bob want to exchange more than one message?

Deterministic Encryption

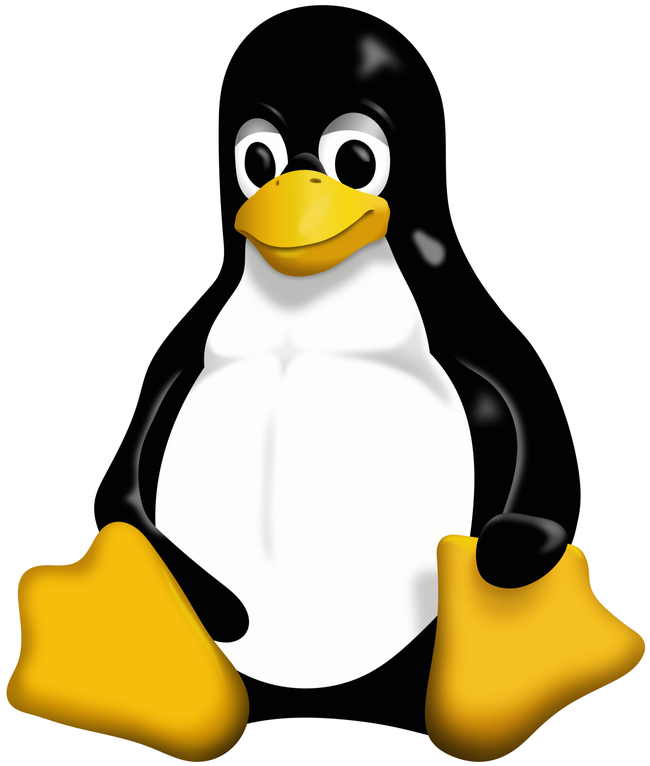
A cipher (Enc, Dec) is **deterministic** if calling $Enc(k, m)$ on the same inputs twice always produces the same output

Deterministic Encryption is (often) Bad



“Tux”

Deterministic Encryption is (often) Bad



"Tux"

"Good" encryption

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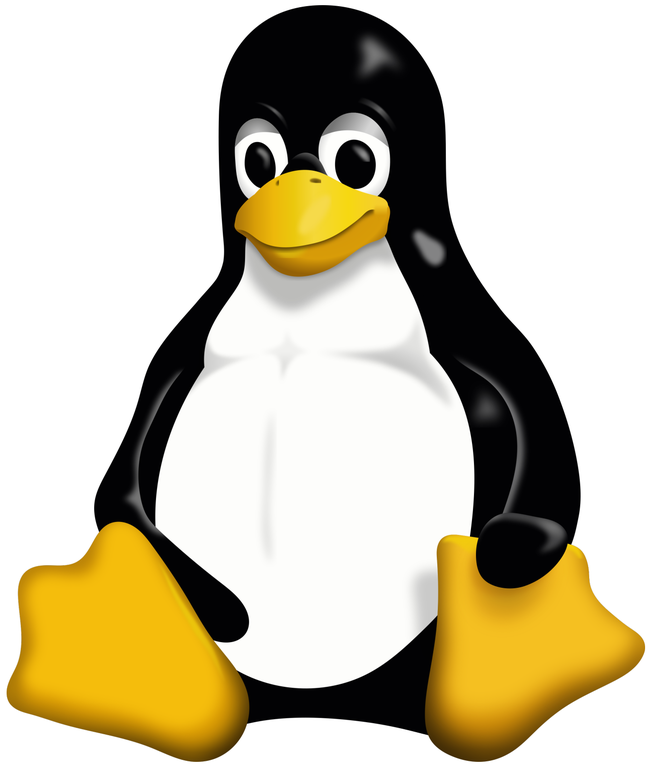

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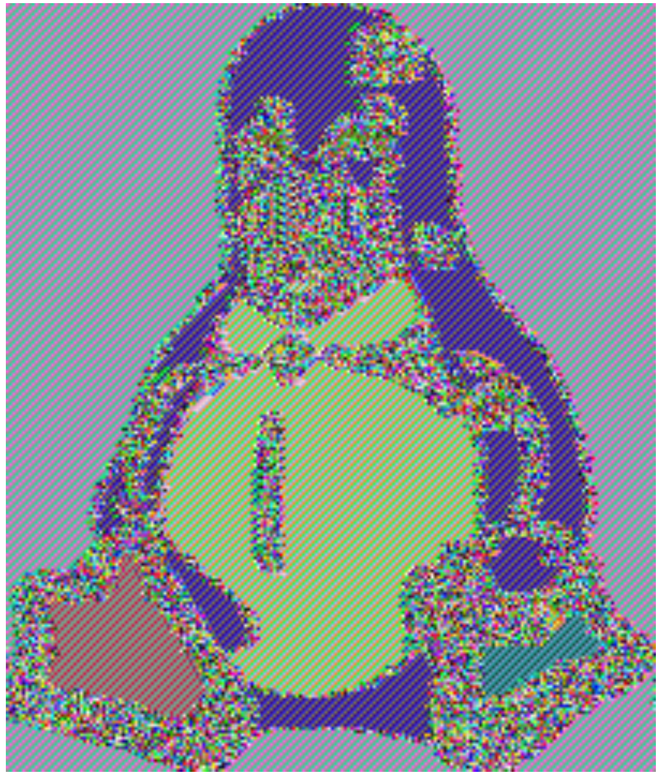

Deterministic Encryption

A cipher (Enc, Dec) has **ciphertext indistinguishability against a chosen plaintext attack (CPA)** if:

Let $Enc_L(k, m_0, m_1) = Enc(k, m_0)$

Let $Enc_R(k, m_0, m_1) = Enc(k, m_1)$

Where m_0, m_1 are of the same length

A cipher (Enc, Dec) has **ciphertext indistinguishability against a chosen plaintext attack (CPA)** if:

Let $Enc_L(k, m_0, m_1) = Enc(k, m_0)$

Let $Enc_R(k, m_0, m_1) = Enc(k, m_1)$

Where m_0, m_1 are of the same length

$$\left\{ Enc_L(k, \cdot, \cdot) \;\middle|\; k \leftarrow K \right\} \approx \left\{ Enc_R(k, \cdot, \cdot) \;\middle|\; k \leftarrow K \right\}$$

A cipher (Enc, Dec) has **random ciphertexts against a chosen plaintext attack (CPA$)** if:

$$Samp(m) = \{c \mid c \leftarrow C(|m|)\}$$

Ciphertext of length corresponding to message m

$$\{Enc(k, \cdot) \mid k \leftarrow K\} \approx Samp(\cdot)$$

In order to achieve CPA security a cipher must not be deterministic

Randomized CPA-Secure Encryption

```
Enc(k, m):
  r ←{0,1}^λ
  c0 = F(k, r) ⊕ m
  c = (c0, r)
  return c

Dec(k, (c0, r)):
  return F(k, r) ⊕ c0
```

Main idea: it is unlikely that Enc will sample the same r more than once

Proof of security is more nuanced here

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